

Currently, I am trying to understand [mathematical games](#) and random structures. I have some pages for logical and combinatorial games linked from this page, and I will be putting up some stuff on random structures in the foreseeable future. The most of this page is devoted to past papers.

I am trying to understand mathematical games --- i.e., those things known as "mathematical games" by logicians and combinatorists, and as "extended games" by game theorists. So what I am doing in this web-site is describing some important research (including my own) with

J. Sym. Logic 54:4 (1989), 1324-

1345. This is the first half of my Ph.D. thesis. Its about abstract recursion on the natural numbers with successor, predecessor, and 0. We find some restrictions on simple (i.e., without parallelism) fixed points. Out of this somewhat esoteric acorn, eventual periodicity (5. below) would grow.

Ann. Pure & Appl.

Logic 48 (1990), 103-134. This is the third fourth of my Ph.D. thesis. Here, we generalize the classical parametrization theorems to non-acceptable structures (which include the finite structures). The discerning reader will detect the blooper on p. 124.

Ann. Pure & Appl. Logic 50 (1990), 129-151. The last fourth of my Ph.D. thesis. Here I launch two conjectures on LFP logic: on a class of (finite) structures,  $FO = LFP$  iff all LFP inductions are bounded, and all LFP inductions are bounded iff the a popular infinitary logic collapses to FO. Kolaitis, Vardi and Dewar confirmed the latter conjecture, while Immerman, Gurevich and Shelah found counterexamples to the former.

J. Comb. Th.-A 57:1 (1991), 68-75. Suppose

that  $P$  is a Cartesian product of posets. Let  $X$  be a coloring of  $P$ : for each tuple  $p$ ,  $X(p)$  is a color in  $I$ . Under what conditions do we know that there must be a color  $i$  such that there is an  $i$ -colored copy of one of the poset factors of  $P$ ? We explore the situation for  $I$  finite and each factor being a finitary tree: such an  $i$  must exist. There is a counterexample when non-finitary trees are allowed. We conjecture that when  $I$  and all factors are finite, such  $i$  must exist.

Notre Dame J. Formal Logic 33:2

(1992), 273-290. Out of the esoteric acorn (see [1], above) comes: on large, chain-like graphs, one-dimensional and monadic second order-definable relations are 'eventually periodic' in the sense that they  $M$   $M$  -de

We look at the game-theoretic version of LFP explored in paper [11] below, and represent dimension and number of variables. We use these characterizations to compute the dimension and number of variables of nonconnectivity.

(with [E. Graedel](#), at the Lehrgebiet Mathematische Grundlagen der Informatik, in Aachen), Inform. & Comp. 119:1 (1995), 129-135. We prove that FO + pos DTC is closed under negation, and within some highly uniform graphs collapses into FO, thus separating it from FO + pos TC. This paper follows a LICS abstract.

J. Sym. Logic 60:2 (1995), 392-413. The long-awaited horrible proof (using eventual periodicity and monotonicity) that non-reachability on finite graphs is precisely 2-dimensional. Remember, you heard it here first.

Inform. & Comp. 122:2 (1995), 201-220. Here, I generalize LFP into a Datalog-like game structure, which allows me to generate a nonlinear hierarchy in stratified least fixed point logic, based on subroutines. This is continued in [8] above, and is the basis for much of my current research. Incidentally, I have since discovered that a more general investigation anticipating some of my approach was launched decades ago by J. Hintikka: see my page on [Game Theoretic Semantics](#) for details.

(with [E. Graedel](#), at the Lehrgebiet Mathematische Grundlagen der Informatik, in Aachen), Ann. Pure & Appl. Logic 77 (1996), 169-199. We prove that non-reachability is not in FO + pos TC. We cook up an alternate hierarchy to the one in [11], and present a proof of (a stronger version of) the Main Theorem of [11]. This paper follows a FOCS abstract.

(with [B. Shekhtman](#) and \_\_\_\_\_)

Godehardt and J. Jaworsky, On the connectivity of a random interval graph, in [Random Structures & Algorithms 9:2, 1996](#), and apparently not reviewed in the AMS Reviews, grump, grump, grump, so let's say that the AMS Review should say that if you want to understand this model, start with this paper.) Among other things, we find that for each fixed  $d$ , the set of a.s. FO sentences in this model is a complete noncategorical theory. This paper follows a LICS abstract.

, Discrete Mathematics 254 (2002) 331-347. We prove a result announced in the AMS abstracts. We use elementary methods to prove that over free labelled trees, MSO definable queries are almost surely true or almost surely false.

, Research on Language and Computation 1 (2003), 203-226. This is an introduction to random trees intended for non-experts (i.e., researchers with no particular background in probabilistic methods or in combinatorics). We describe three useful techniques --- moment methods, generating functions, and branching processes. We also give some background and applications to computation.

, Bulletin of the Institute of Combinatorics and its Applications 38 (2003) 84-100. After my paper on , I conjectured that any 2-coloring (i.e., red & blue) of a Cartesian product of finite posets  $P \times Q$  admits either a red copy of  $P$  or a blue copy of  $Q$ . In this paper, I disprove this conjecture by proving that if  $n$  is sufficiently larger than  $m$ , then there is a 2-coloring of the poset of the power set of  $[m+n]$  that does not admit either a red copy of the poset of the power set of  $[n]$ , nor a blue copy of the poset of the power set of  $[2]$ .

, J. Logic, Language and Information, to appear. This article introduces a (positive) least fixed point logic in which quantification is restricted by guard relations. This logic is more relaxed than previous guarded least fixed point logics, which were motivated by modal logic type considerations: this one was motivated by game logic and database considerations. This logic turns out to have the same expressive power as the classical positive least fixed point logic of Moschovakis.

[20. On the Structure of Random Unlabelled Acyclic Graphs](#), Discrete Mathematics 227 (2004), 147 - 170. We use prove a variation of a result of [Alan Woods](#) (that all MSO queries over random trees have asymptotic probabilities, which he proved using hardcore generating function methods: see RSA 10 (1997) Colouring rules for finite trees ...) that over free unlabelled trees, MSO definable queries are almost surely true or almost surely false. The proof is an elementary version of Woods's approach, and this scenic route gives us a lot of information about the anatomy of (almost all) unlabelled trees.

, Combinatorics, Probability, Computing 13 (2004), 373 - 387. This article presents a proof that in the 1-dimensional model of Gilbert random graphs, all upwards closed properties have at least weak thresholds. In addition, all upwards closed properties whose thresholds are sufficiently higher than the threshold for connectivity have strong thresholds. We also present some counterexamples. I have just seen an

interesting paper on a variant of these problems for higher dimensions by [Ashish Goel](#), Bhaskar Krishnamacari, and Sanatan Rai; they use matching methods which, alas, will suffice for the not-so-sparse strong threshold results but not for the just-above-phase-transition weak threshold results.

[Inductive Norms and Negation](#), in preparation.

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